Lecture 4

5. HARMONIC CURRENT CIRCUIT CALCULATION

As a rule, calculation of electric circuits includes the following stages:

1) representation of a real electric circuit by means of a theoretical model — an equivalent electric circuit;

2) building a mathematical model — deriving a system of linearly independent equations, the unknown variables in which are the voltages and currents in the circuit — electrical equilibrium equations;

3) solving the system of electrical equilibrium equations and bringing the obtained results in compliance with the model being calculated.

At the first stage, each real element of an electric circuit (a generator — a source of energy, an electric motor, a resistor, a capacitor, etc.) is replaced by its simplified model consisting of only idealized passive and active elements. At the second stage, a system of electrical equilibrium equations is derived on the basis of the accepted system of independent variables that describe the electromagnetic processes in the circuit under study. Finally, in the third stage, this system is solved and the response of the circuit to the external stimulus is determined, or expressions relating these values are written.

5.1. The Basic Laws of Electrical Engineering

The general form of Ohm's law (1.8) for an electric circuit can be written as

$$i = \frac{u}{Z}, \tag{5.1}$$

where i, u, Z are the instantaneous values of current and voltage and the impedance of the circuit respectively.

There are also two Kirchhoff's laws.

Kirchhoff's current law: the sum of the currents in a node is zero.

$$\sum_{k=1}^{n} i_k = 0, \tag{5.2}$$

where i_k — instantaneous value of the current in the k-th branch converging to a given node; n — total number of branches converging to this node.

So, for node 2 in Fig. 2.1 we can write:

$$i_2 - i_4 - i_3 - j_1 = 0,$$

Here the currents directed to the node (i_2) have a "plus" sign and those directed from the node (i_4, i_5, j_1) have a "minus" sign.

Kirchhoff's current law (5.2) can be formulated for a cross-section as well: the sum of the currents in a cross-section is equal to zero. Then in (5.2) i — current of the k-th branch and n — number of branches in this cross-section. So, for the cross-section $q-q$ in Fig. 2.3, c we can write:

$$-i_1+i_3-i_4-i_5-j_1=0.$$

Here the currents whose directions coincide with the direction of the cross-section (i_3) have a "plus" sign whereas the currents whose directions are opposite to the direction of the cross-section (i_1, i_4, i_5, j_1) have a "minus" sign.

Kirchhoff's current law for a given circuit can be written by means of topological matrices. So, using the topological matrix (2.2), we can write for the circuit in Fig. 2.1:

$$\mathbf{A}_a i=0,$$

where $\boldsymbol{i}$ — matrix-column of currents in the branches:

$$\boldsymbol{i}=\begin{bmatrix} i_1\\ i_2\\ i_3\\ i_4\\ i_5\\ i_6\\ j_1 \end{bmatrix}$$

or

$$\begin{bmatrix} 1 & 1 & -1 & 0 & 0 & 0 & 0\\ 0 & -1 & 0 & 1 & 1 & 0 & 1\\ 0 & 0 & 1 & 0 & -1 & 1 & -1\\ -1 & 0 & 0 & -1 & 0 & -1 & 0 \end{bmatrix}\begin{bmatrix} i_1\\ i_2\\ i_3\\ i_4\\ i_5\\ i_6\\ j_1 \end{bmatrix}=\begin{bmatrix} i_1+i_2-i_3\\ -i_2+i_4+i_5+j_1\\ i_3-i_5+i_6-j_1\\ -i_1-i_4-i_6 \end{bmatrix}=0.$$

Using the matrix of cross-sections $\mathbf{D}_a$ (2.12), we can write for the circuit in Fig. 2.1:

$$\mathbf{D}_a i=0,$$

or

$$\begin{bmatrix} -1 & -1 & 1 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & -1 & -1 & 0 & -1\\ 0 & 0 & -1 & 0 & 1 & -1 & 1\\ 1 & 0 & 0 & 1 & 0 & 1 & 0\\ 0 & 1 & -1 & -1 & 0 & -1 & 0\\ -1 & 0 & 1 & -1 & -1 & 0 & -1\\ 1 & 1 & 0 & 0 & -1 & 1 & -1 \end{bmatrix}\begin{bmatrix} i_1\\ i_2\\ i_3\\ i_4\\ i_5\\ i_6\\ j_1 \end{bmatrix}=\begin{bmatrix} -i_1-i_2+i_3\\ i_2-i_4-i_5-j_1\\ -i_3+i_5-i_6+j_1\\ i_1+i_4+i_6\\ i_2-i_3-i_4-i_6\\ -i_1+i_3-i_4-i_5-j_1\\ i_1+i_2-i_5+i_6-j_1 \end{bmatrix}=0.$$

Here Kirchhoff's current law is written for the sections: *k-k, l-l, m-m, n-n, p-p, g-g, r-r* of the circuit graph in Fig. 2.3.

Kirchhoff's voltage law: the sum of the voltages in the loop is equal to zero

$$\sum_{k=1}^{n} u_k=0, \qquad (5.3)$$

where u_k — instantaneous value of the voltage in the k-th branch belonging to a given loop; n — total number of branches of the loop.

So, for the loop $r_1-e_1-r_2-r_4$ of the circuit in Fig. 2.1, we can write, bypassing the loop in the clockwise direction

$$-u_1-e_1+u_2+u_4=0.$$

Here the voltages whose directions coincide with the direction of tracing the loop (u_2, u_4) have a "plus" sign, and the voltages whose directions are opposite to the direction of tracing the loop (u_1, e_1) have a "minus" sign.

Kirchhoff's voltage law for a given circuit can be written by means of topological matrices. So, using the loop matrix $\mathbf{B}_a$ (2.5), we can write for the circuit in Fig. 2.1:

$$\mathbf{B}_a u=0,$$

where $\boldsymbol{u}$ — matrix-column of the voltages in the branches:

$$\boldsymbol{u} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ e_1 \end{bmatrix}.$$

Here the path to the e_1 voltage source is shown as a separate branch. Then we get

$$\begin{bmatrix} -1 & 1 & 0 & 1 & 0 & 0 & -1 \\ -1 & 1 & 0 & 0 & 1 & 1 & -1 \\ -1 & 0 & -1 & 0 & 0 & 1 & 0 \\ -1 & 0 & -1 & 1 & -1 & 0 & 0 \\ 0 & -1 & -1 & 0 & -1 & 0 & 1 \\ 0 & -1 & -1 & -1 & 0 & 1 & 1 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \\ e_1 \end{bmatrix} = \begin{bmatrix} -u_1 + u_2 + u_4 - e_1 \\ -u_1 + u_2 + u_5 + u_6 - e_1 \\ -u_1 - u_3 + u_6 \\ -u_1 - u_3 + u_4 - u_5 \\ -u_2 - u_3 - u_5 + e_1 \\ -u_2 - u_3 - u_4 + u_5 + e_1 \\ -u_4 + u_5 + u_6 \end{bmatrix} = 0.$$

Here Kirchhoff's voltage law is written for the loops: $r_1 - e_1 - r_2 - r_4$, $r_1 - e_1 - r_2 - r_5 - r_6$, $r_1 - r_3 - r_6$, $r_2 - e_1 - r_3 - r_5$, $r_2 - e_1 - r_3 - r_6 - r_4$, $r_4 - r_5 - r_6$ of the circuit in Fig. 2.1.

In harmonic current circuit calculations Ohm's laws and Kirchhoff's laws are written in complex form.

For expression (5.1) the image can be written in terms of complex amplitudes:

$$\dot{I}_m = \frac{\dot{U}_m}{Z} = Y\dot{U}_m.$$

This is the way Ohm's law is written in complex form.

For expression (5.2) the image is written in terms of complex amplitudes as:

$$\sum_{k=1}^{n} \dot{I}_{mk} = 0.$$

This is the way Kirchhoff's current law is written in complex form.

For expression (5.3) the image is written in terms of complex amplitudes as:

$$\sum_{k=1}^{n} \dot{U}_{mk} = 0.$$

This is the way Kirchhoff's voltage law is written in complex form.

5.2. Equivalent Transformations in Electric Circuits

Two sections of an electric circuit are called equivalent if replacement of one of them by the other does not change the currents and voltages in the rest of the circuit.

When calculating an electric circuit, it is necessary to build a system of linearly independent equations of electrical equilibrium and solve it. Equivalent transformation of an electric circuit is based on equivalent transformation of the corresponding system of electrical equilibrium equations.

5.2.1. Series Connection of Elements

Consider an electric circuit with a serial connection of active resistances $r_1, r_2, ..., r_k$, inductances $L_1, L_2, ..., L_m$, capacitances $C_1, C_2, ..., C_n$, and voltage sources with the EMF $e_1, e_2, ..., e_v$, (Fig. 5.1).

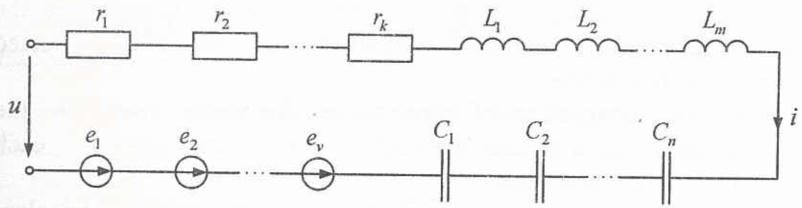


Fig. 5.1

According to Kirchhoff's voltage law, we get for the instantaneous values:

$$ir_1 + ir_2 + ... + ir_k + L_1\frac{di}{dt} + L_2\frac{di}{dt} + ... + L_m\frac{di}{dt} +$$

$$+\frac{1}{C_1}\int_0^t idt + \frac{1}{C_2}\int_0^t idt + ... + \frac{1}{C_n}\int_0^t idt + e_1 + e_2 + ... + e_v - u = 0$$

or

$$i(r_1+r_2+\ldots+r_k)+(L_1+L_2+\ldots+L_m)\frac{di}{dt}+$$

$$+\left(\frac{1}{C_1}+\frac{1}{C_2}+\ldots+\frac{1}{C_n}\right)\int_0^t idt+(e_1+e_2+\ldots+e_v)=u$$

and finally:

$$ir_{eq}+L_{eq}\frac{di}{dt}+\frac{1}{C_{eq}}\int_0^t idt+e_{eq}=u,$$

where

$$r_{eq}=(r_1+r_2+\ldots+r_k)=\sum_{i=1}^{k} r_i,$$

$$L_{eq}=(L_1+L_2+\ldots+L_m)=\sum_{i=1}^{m} L_i,$$

$$\frac{1}{C_{eq}}=\left(\frac{1}{C_1}+\frac{1}{C_2}+\ldots+\frac{1}{C_n}\right)=\sum_{i=1}^{n}\frac{1}{C_i},$$

$$e_{eq}=(e_1+e_2+\ldots+e_v)=\sum_{i=1}^{v} e_i.$$

That is, for a series connection of resistances or inductances the equivalent resistance or inductance equals the sum of series-connected resistances or inductances.

For a series connection of capacitances the value inverse to the equivalent capacitance equals the sum of the inverse values of each series-connected capacitances.

For a series connection of voltage sources the value of the equivalent voltage source equals the algebraic sum of the values of each series-connected voltage source.

A series connection of ideal current sources is impossible.

Similar relationships can be obtained for complex resistances and EMFs:

$$Z_{eq}=\sum_{i=1}^{k} Z_{ri}+\sum_{i=1}^{m} Z_{Li}+\sum_{i=1}^{n} Z_{Ci}=\sum_{i=1}^{i} Z_i;\; \dot{E}_{meq}=\sum_{i=1}^{v}\dot{E}_{mi}.$$

The rules of series connection of elements constitute the basis for a voltage divider (Fig. 5.2).

Here

$$\dot{U}_{ml}=\frac{\dot{U}_m Z_1}{Z_1+Z_2};\; \dot{U}_{m2}=\frac{\dot{U}_m Z_2}{Z_1+Z_2}.$$

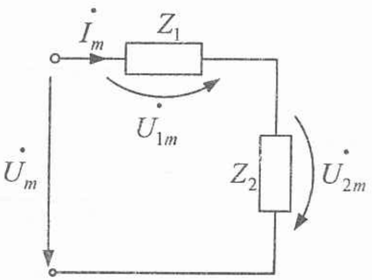


Fig. 5.2

5.2.2. Parallel Connection of Elements

Consider an electric circuit with a parallel connection of resistances $r_1, r_2, \ldots, r_k$, inductances $L_1, L_2, \ldots, L_m$, capacitances $C_1, C_2, \ldots, C_n$, and current sources $j_1, j_2, \ldots, j_v$, (Fig. 5.3).

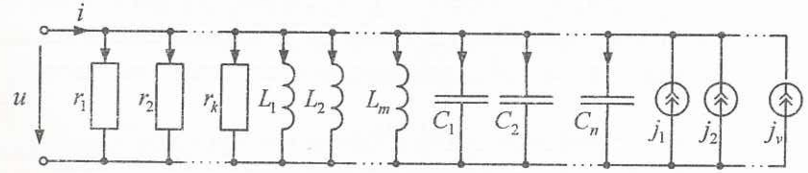


Fig. 5.3

According to Kirchhoff's current law, we get for the instantaneous values:

$$i-\frac{u}{r_1}-\frac{u}{r_2}-\ldots-\frac{u}{r_k}-\frac{1}{L_1}\int_0^t ud\tau-\frac{1}{L_2}\int_0^t ud\tau-\ldots-$$

$$-\frac{1}{L_m}\int_0^t ud\tau-\ldots-\frac{1}{L_m}\int_0^t ud\tau-C_1\frac{du}{dt}-C_2\frac{du}{dt}-\ldots-$$

$$-C_n\frac{du}{dt}+j_1+j_2+\ldots+j_v=0$$

or

$$i-\left(\frac{1}{r_1}+\frac{1}{r_2}+\ldots+\frac{1}{r_k}\right)u-\left(\frac{1}{L_1}+\frac{1}{L_2}+\ldots+\frac{1}{L_m}\right)\int_0^t ud\tau-$$

$$-(C_1+C_2+\ldots+C_n)\frac{du}{dt}+(j_1+j_2+\ldots+j_v)=0$$

and finally

$$i-\frac{1}{r_{eq}}u-\frac{1}{L_{eq}}\int_0^t u d\tau - C_{eq}\frac{du}{dt}+j_{eq}=0,$$

where

$$\frac{1}{r_{eq}}=\frac{1}{r_1}+\frac{1}{r_2}+...+\frac{1}{r_k}=\sum_{i=1}^{k}\frac{1}{r_i},$$

$$\frac{1}{L_{eq}}=\frac{1}{L_1}+\frac{1}{L_2}+...+\frac{1}{L_m}=\sum_{i=1}^{m}\frac{1}{L_i},$$

$$C_{eq}=C_1+C_2+...+C_n=\sum_{i=1}^{n}C_i,$$

$$j_{eq}=j_1+j_2+...+j_\nu=\sum_{i=1}^{\nu}j_i.$$

That is, for a parallel connection of resistances or inductances the value of the inverse equivalent resistance or inductance equals the sum of the inverse values of each parallel-connected resistance or inductance.

For a parallel connection of capacitances the equivalent capacitance equals the sum of parallel-connected capacitances.

For a parallel connection of current sources the value of the equivalent current source equals the algebraic sum of the values of each parallel-connected current source.

A parallel connection of ideal voltage sources is impossible.

Similar relationships can be obtained for complex conductances and current sources.

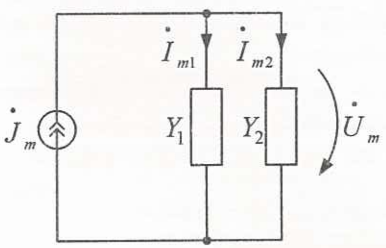


Fig. 5.4

$$Y_{eq}=\sum_{i=1}^{k}Y_{ri}+\sum_{i=1}^{m}Y_{Li}+\sum_{i=1}^{n}Y_{Ci}=\sum_{i=1}^{l}Y_i;$$

$$\dot I_{meq}=\sum_{i=1}^{\nu}\dot I_{mi}.$$

The rules of parallel connection of elements constitute the basis for a current divider (Fig. 5.4).

Here

$$\dot I_{m1}=\frac{\dot I_m Y_1}{Y_1+Y_2};\ \dot I_{m2}=\frac{\dot I_m Y_2}{Y_1+Y_2} \qquad (5.4)$$

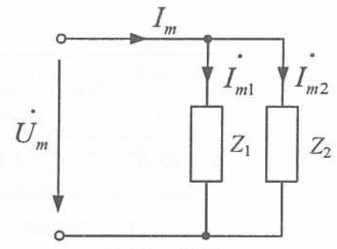


Fig. 5.5

or, in terms of resistances (Fig. 5.5):

Here

$$\dot I_{m1}=\frac{\dot I_m Z_1}{Z_1+Z_2};\ \dot I_{m2}=\frac{\dot I_m Z_2}{Z_1+Z_2} \qquad (5.5)$$

Expressions (5.4) and (5.5) represent the rule of "*the alien resistance*": the current in one of two parallel-connected resistances equals the total current divided by the sum of these resistances and multiplied by the other "*alien*" resistance.

5.2.3. Mutual Equivalent Transformations of a Parallel and Series Connections of Elements

The problem of equivalent transformation of a series connection of elements of the same type to a parallel connection and vice versa is ambiguous. Let us consider transformation of a series and a parallel connection of elements of various types in a harmonic current circuit.

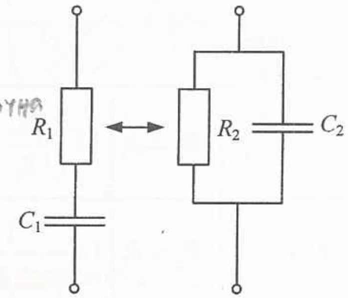


Fig. 5.6

In Fig. 5.6 are shown equivalent series and parallel connections of elements.

Obviously, we can write:

$$Z_1=R_1+\frac{1}{j\omega C_1}=Z_2=\frac{R_2\frac{1}{j\omega C_2}}{R_2+\frac{1}{j\omega C_2}}.$$

Hence,

$$Z_2=\frac{R_2}{1+j\omega R_2C_2}=\frac{R_2(1-j\omega R_2C_2)}{(1+j\omega R_2C_2)(1-j\omega R_2C_2)}=\frac{R_2-j\omega R_2^2C_2}{1+\omega^2R_2^2C_2^2}=$$

$$=\frac{R_2}{1+\omega^2R_2^2C_2^2}-j\frac{\omega R_2^2C_2}{1+\omega^2R_2^2C_2^2}=R_1+\frac{1}{j\left(\frac{1}{\omega R_2^2C_2}+\omega C_2\right)}=$$

$$=R_1+\frac{1}{j\omega C_2\left(1+\frac{1}{\omega^2R_2^2C_2}\right)}=R_1+\frac{1}{j\omega C_1}.$$

That is

$$R_1=\frac{R_2}{1+\omega^2R_2^2C_2^2};\quad C_1=C_2+\frac{1}{\omega^2R_2^2C_2}.$$

Relationships for other elements can be obtained in a similar way. Table 5.1 presents relationships for equivalent transformation of the series connection of the elements R_1, L_1, C_1 to the parallel connection of the elements R_2, L_2, C_2 and vice versa.

Table 5.1

Original circuit	Transformed circuit
R_1, L_1	$R_2=R_1\left[1+\left(\frac{\omega L_1}{R_1}\right)^2\right];\ L_2=L_1\left[1+\frac{R_1^2}{(\omega L_1)^2}\right]$
R_1, C_1	$R_2=R_1\left[1+\frac{1}{(\omega R_1C_1)^2}\right];\ C_2=\frac{C_1}{1+(\omega R_1C_1)^2}$
R_2, L_2	$R_1=\frac{R_2}{1+\left(\frac{R_2}{\omega L_2}\right)^2};\ L_1=\frac{L_2}{1+\left(\frac{\omega L_2}{R_2}\right)^2}$
R_2, C_2	$R_1=\frac{R_2}{1+(\omega R_2C_2)^2};\ C_1=C_2\left[1+\frac{1}{(\omega R_2C_2)^2}\right]$

The problem of equivalent transformation of the series connection of the elements L_1, C_1 to the parallel connection of L_2, C_2 is also ambiguous.

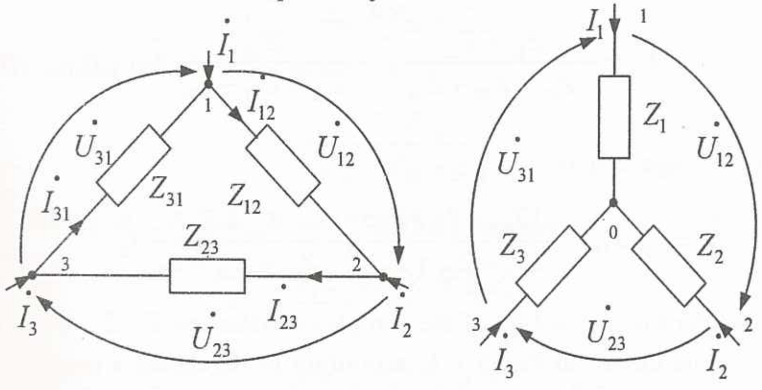
It should note that the expressions in Table 5.1 for the transformed circuit are valid for the same frequency.

If you change the frequency, the values of the parameters of the transformed circuit change.

It follows that the mutual equivalent transformations of series and parallel connections of various types of elements are impossible for nonlinear circuits, or for linear non-harmonic current circuits.

5.2.4. Transformation of Delta- to Star-Connection and Vice Versa

The connections of elements in Fig. 5.7, *a* and *b* are called the delta- and the star-connection respectively.



a *b*

Fig. 5.7

Consider the connection of the complex resistances Z_{12}, Z_{23}, Z_{31} in the triangle (delta connection). For the network in Fig. 5.7, *a*, according to Kirchhoff's current and voltage laws, the following expressions can be written for nodes 1, 2 and the loop $Z_{12}-Z_{23}-Z_{31}$:

$$\begin{cases}\dot I_1+\dot I_{31}-\dot I_{12}=0;\\ \dot I_2+\dot I_{12}-\dot I_{23}=0;\\ \dot I_{12}Z_{12}+\dot I_{23}Z_{23}+\dot I_{31}Z_{31}=0.\end{cases}$$

Substituting the current $\dot{I}_{12}$ from the first equation to the other equations, we get

$$\begin{cases} \dot{I}_1 + \dot{I}_2 - \dot{I}_{23} + \dot{I}_{31} = 0; \\ \dot{I}_1 Z_{12} + \dot{I}_{23} Z_{23} + \dot{I}_{31}(Z_{12} + Z_{31}) = 0. \end{cases}$$

Substituting the current $\dot{I}_{23}$ from the first equation to the second one, we get

$$\dot{I}_1(Z_{12} + Z_{23}) + \dot{I}_2 Z_{23} + \dot{I}_{31}(Z_{12} + Z_{23} + Z_{31}) = 0.$$

Hence,

$$\dot{I}_{31} = -\frac{Z_{12} + Z_{23}}{Z_{12} + Z_{23} + Z_{31}}\dot{I}_1 - \frac{Z_{23}}{Z_{12} + Z_{23} + Z_{31}}\dot{I}_2.$$

The voltage $\dot{U}_{31}$ is:

$$\dot{U}_{31} = \dot{I}_{31} Z_{31} = -\frac{(Z_{12} + Z_{23})Z_{31}}{Z_{12} + Z_{23} + Z_{31}}\dot{I}_1 - \frac{Z_{23} * Z_{31}}{Z_{12} + Z_{23} + Z_{31}}\dot{I}_2. \qquad (5.6)$$

Consider the connection of the complex resistances Z_1, Z_2, Z_3 in the star. For the circuit in Fig. 5.7, *b*, according to Kirchhoff's current and voltage laws, the following expressions can be written for node "0" and the loop $Z_1 - Z_3 - U_{31}$:

$$\begin{cases} \dot{I}_1 + \dot{I}_2 + \dot{I}_3 = 0; \\ \dot{I}_1 Z_1 - \dot{I}_3 Z_3 + \dot{U}_{31} = 0. \end{cases}$$

Substituting the current $\dot{I}_3$ from the first equation to the second equation, we get

$$\dot{I}_1(Z_1 + Z_3) + \dot{I}_2 Z_3 + \dot{U}_{31} = 0.$$

From here

$$\dot{U}_{31} = -(Z_1 + Z_3)\dot{I}_1 - Z_3\dot{I}_2. \qquad (5.7)$$

As in equivalent transformations the currents $\dot{I}_1, \dot{I}_2, \dot{I}_3$ and voltages $\dot{U}_{12}, \dot{U}_{23}, \dot{U}_{31}$ are the same in both networks (Fig. 5.7, *a* and Fig. 5.7, *b*), from (5.6) and (5.7) we get:

$$Z_1 + Z_3 = \frac{(Z_{12} + Z_{23})Z_{31}}{Z_{12} + Z_{23} + Z_{31}}, \qquad (5.8)$$

$$Z_3 = \frac{Z_{23}Z_{31}}{Z_{12} + Z_{23} + Z_{31}}. \qquad (5.9)$$

Substituting (5.9) into (5.8), we get

$$Z_1 = \frac{Z_{31}Z_{12}}{Z_{12} + Z_{23} + Z_{31}}. \qquad (5.10)$$

By similar transformations we have

$$Z_2 = \frac{Z_{12}Z_{23}}{Z_{12} + Z_{23} + Z_{31}}. \qquad (5.11)$$

Defining Z_{12}, Z_{23}, Z_{31} from (5.9)–(5.11) we get:

$$Z_{12} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_3}, \qquad (5.12)$$

$$Z_{23} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_1}, \qquad (5.13)$$

$$Z_{31} = \frac{Z_1Z_2 + Z_2Z_3 + Z_3Z_1}{Z_2}. \qquad (5.14)$$

Replacing resistances by conductances in (5.9)–(5.11), we get:

$$Y_1 = \frac{Y_{12}Y_{23} + Y_{23}Y_{31} + Y_{31}Y_{12}}{Y_{23}}, \qquad (5.15)$$

$$Y_2 = \frac{Y_{12}Y_{23} + Y_{23}Y_{31} + Y_{31}Y_{12}}{Y_{31}}, \qquad (5.16)$$

$$Y_3 = \frac{Y_{12}Y_{23} + Y_{23}Y_{31} + Y_{31}Y_{12}}{Y_{12}}. \qquad (5.17)$$

We can see that relationships (5.15)–(5.17) for the conductances of the star-connection are identical in structure to relationships (5.12)–(5.14) for the resistances of the delta-connection. Obviously, we should expect that relationships (5.9)–(5.11) for the resistances of the star-connection have their related relationships for the conductances of the delta-connection that have the same structure. Indeed, replacing resistances by conductances in (5.12)–(5.14), we get:

$$Y_{12} = \frac{Y_1 Y_2}{Y_1 + Y_2 + Y_3}, \tag{5.18}$$

$$Y_{23} = \frac{Y_2 Y_3}{Y_1 + Y_2 + Y_3}, \tag{5.19}$$

$$Y_{31} = \frac{Y_3 Y_1}{Y_1 + Y_2 + Y_3}. \tag{5.20}$$

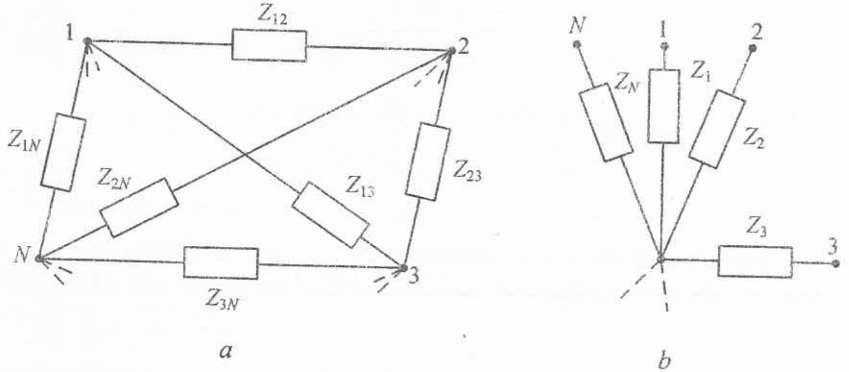


Fig. 5.8

The obtained relationships apply to both a delta and to a three-beam star-connection (Y-network). In the general case, transforming an N-beam star-connection to an N-angular network (Fig. 5.8, *a*, *b*), we get similarly to (5.18)–(5.20):

$$Y_{kL} = \frac{Y_k Y_L}{Y_1 + Y_2 + \ldots + Y_N}.$$

An inverse conversion of the N-angular network to the N-beam star connection is impossible in the general case.

5.2.5. Transformation of Circuits with Ideal Voltage and Current Sources

As both voltage and current sources are sources of energy, they can be mutually transformed. So, for a real voltage source (see Fig. 1.4, *b*) and a real current source (see Fig. 1.6, *b*), expressing the current from (1.32), the same currents "i" and voltages "u" being the same, we get

$$i = \frac{e-u}{r_i}.$$

Equating it to (1.33), we obtain

$$j = \frac{e}{r_i} \text{ while } g_i = \frac{1}{r_i}.$$

Expressing the voltage from (1.33):

$$u = \frac{j-i}{g_i}$$

and equating it to (1.32), we obtain

$$e = \frac{j}{g_i} \text{ while } r_i = \frac{1}{g_i}.$$

There are also methods for transferring ideal voltage and current sources. Consider Fig. 5.9. Let us derive equations for the network in Fig. 5.9, *a* using Kirchhoff's laws.

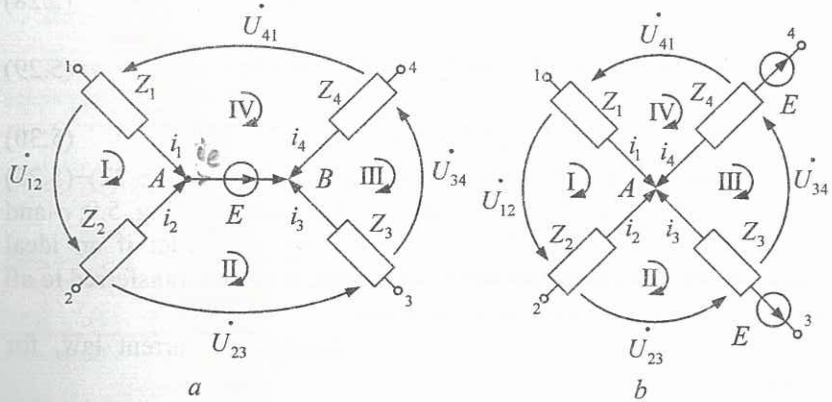


Fig. 5.9

For nodes A and B we get:

$$\dot{I}_1 + \dot{I}_2 - \dot{I}_E = 0, \quad \dot{I}_E + \dot{I}_3 + \dot{I}_4 = 0.$$

After summation:

$$\dot{I}_1+\dot{I}_2+\dot{I}_3+\dot{I}_4=0. \quad (5.21)$$

For loops I–IV we get:

$$\dot{I}_1\cdot Z_1-\dot{I}_2\cdot Z_2-\dot{U}_{12}=0, \quad (5.22)$$

$$\dot{I}_2\cdot Z_2-\dot{E}-\dot{I}_3\cdot Z_3-\dot{U}_{23}=0, \quad (5.23)$$

$$\dot{I}_3\cdot Z_3-\dot{I}_4\cdot Z_4-\dot{U}_{34}=0, \quad (5.24)$$

$$\dot{I}_4\cdot Z_4+\dot{E}-\dot{I}_1\cdot Z_1-\dot{U}_{41}=0. \quad (5.25)$$

Make an equation for the circuit in Fig. 5.9, *b*.
For the node *A* we get:

$$\dot{I}_1+\dot{I}_2+\dot{I}_3+\dot{I}_4=0. \quad (5.26)$$

For the loops I–IV we obtain:

$$\dot{I}_1 Z_1-\dot{I}_2 Z_2-\dot{U}_{12}=0; \quad (5.27)$$

$$\dot{I}_2 Z_2-\dot{E}-\dot{I}_3 Z_3-\dot{U}_{23}=0; \quad (5.28)$$

$$\dot{E}+\dot{I}_3 Z_3-\dot{I}_4 Z_4-\dot{E}-\dot{U}_{34}=0; \quad (5.29)$$

$$\dot{I}_4 Z_4+\dot{E}-\dot{I}_1 Z_1-\dot{U}_{41}=0. \quad (5.30)$$

Hence, we can see that equations (5.21) and (5.26), (5.22)–(5.25) and (5.27)–(5.30) are identical. That is the networks in Fig. 5.9, *a* and Fig. 5.9, *b* are equivalent. As a result we can conclude: if an ideal voltage source is included between two nodes, it can be transferred to all branches proceeding from one of the nodes.

Consider Fig. 5.10, *a*. According to Kirchhoff's current law, for nodes *A*, *B*, *C* we get:

$$\dot{I}_1+\dot{J}-\dot{I}_4=0; \quad (5.31)$$

$$\dot{I}_2+\dot{I}_4-\dot{I}_5=0; \quad (5.32)$$

$$\dot{I}_3+\dot{I}_5-\dot{J}=0. \quad (5.33)$$

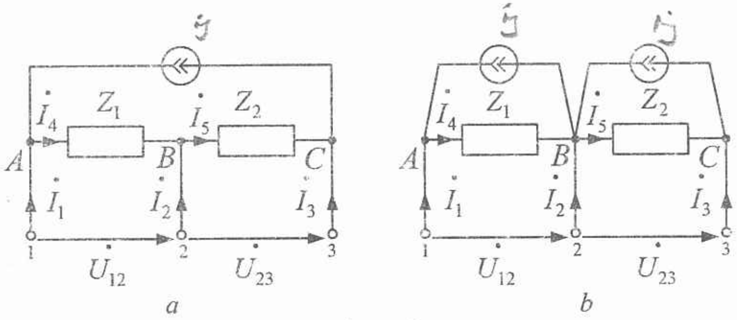


Fig. 5.10

For the network in Fig. 5.10, *b* the following expressions can be written for nodes *A*, *B*, *C*:

$$\dot{I}_1+\dot{J}-\dot{I}_4=0, \quad (5.34)$$

$$\dot{I}_2+\dot{I}_4-\dot{J}-\dot{I}_5+\dot{J}=0, \quad (5.35)$$

$$\dot{I}_3+\dot{I}_5-\dot{J}=0. \quad (5.36)$$

We can see that equations (5.31)–(5.33) and (5.34)–(5.36) are identical. Therefore, the networks in Fig. 5.10, *a* and Fig. 5.10, *b* are equivalent. As a result, the conclusion can be made: if an ideal current source is included between two nodes, it can be moved in parallel to all branches that form a path between these nodes.

Example 1

Determine the complex input impedance and the parameters of the equivalent circuits (Fig. 5.11, *a*, *b*). The parameters of the elements are:

$$C_1=70\text{ pF};\ C_2=30\text{ pF};\ C_3=200\text{ pF};\ C_4=100\text{ pF};$$

$$C_5=C_6=C_7=300\text{ pF};\ L_1=L_2=L_3=\dots=L_6=8\text{ mH}.$$

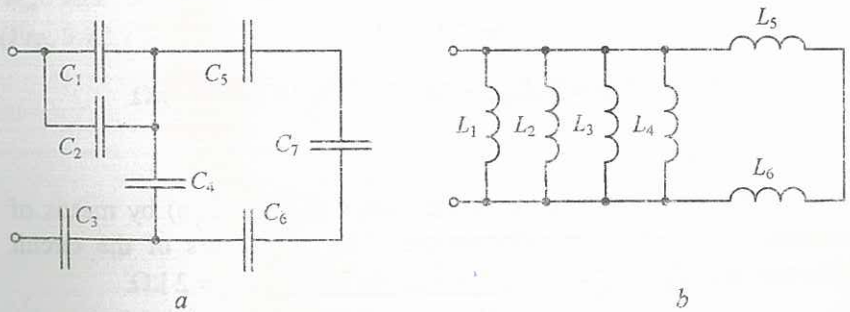


Fig. 5.11